Experimental Study of Transverse and Longitudinal Standing Waves

1. Aim of the laboratory

To observe the transverse (in a wire) and longitudinal (in a spring) standing waves. To calculate the wavelength and frequency corresponding to fundamental and superior harmonics modes of vibration in both cases, and to compare the experimental with theoretic values.

2. General considerations

When mechanical waves are confined in space, as they are with a piano string or an organ pipe, there are reflections at both ends, and waves travel in both directions. They combine according to the general law of wave interference. For a given system (string, pipe, tube, etc.) there are certain frequencies for which the interference results in a stationary vibration pattern called a standing wave. The study of standing waves has many applications in the field of music and in nearly all areas of science and technology. First, we will consider the standing waves produced in a stretched string, which are easily visualized. Then, we will then consider the longitudinal standing waves excited in spring.

3. Transverse waves. Theoretical approach

Only solid media are capable to transmit transverse mechanical waves. For a 1D medium (a tensioned string), the differential equation describing the propagation of transverse waves gives:

$$\frac{\partial^2\psi}{\partial x^2} = \frac{\mu}{F}\cdot\frac{\partial^2\psi}{\partial t^2} = \frac{1}{v^2}\cdot\frac{\partial^2\psi}{\partial t^2} \qquad (1)$$

where $\psi(x,t)$ is the transverse deformation of the string, F is the force which tensions the string, μ is the linear density (mass per unit length) of the string and v is the speed of transverse waves along a string. The solution of the equation (1) considers both the progressive and regressive waves:

$$\psi(x,t) = \psi_p + \psi_r = A\cdot\sin(\omega t + x) + A\cdot\sin(\omega t + kx + \pi), \qquad (2)$$

and:

$$\psi(x,t) = 2\cdot A\cdot\sin kx\cdot\cos(\omega t) = A(x)\cdot\cos(\omega t). \qquad (2')$$

The amplitude A(x) = 2Asin(kx) is constant in time for a fixed position x along the string and for certain frequencies of the mechanical oscillation (for the eigen frequencies of the oscillating system). A minimum value, A(x) = 0 corresponds to the nodes positions, for which kx = nπ and thus:

$$x = n\cdot\frac{\pi}{k} = n\cdot\frac{\lambda}{2}\text{ with } (n = 1,2,..). \qquad (3)$$

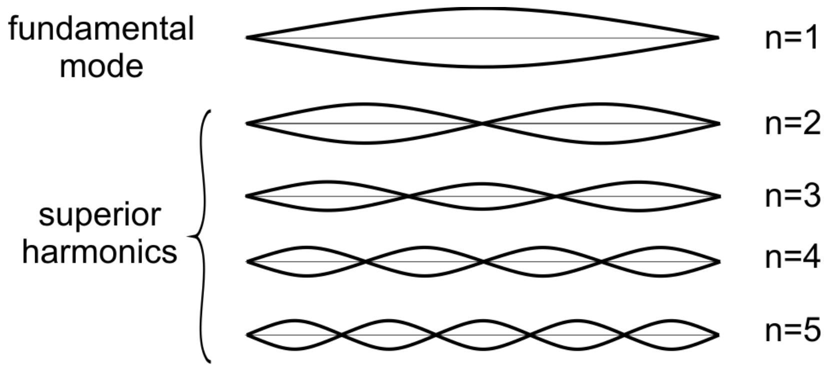

Figure 1

The maximum values of the amplitude A(x) correspond to the antinodes, where A_{max} = 2A, for $kx = (2n+1)\pi/2$, or $x = (2n+1)\lambda/4$.

One has to remark that the energy of standing waves remains stationary; it cannot be transmitted across the nodes, which do not vibrate. The fixed end of the string is certainly a node. Standing wave patterns such as those shown in figure 1 are produced at certain frequencies of the mechanical oscillations, the eigen (characteristic, proper) frequencies of the string. The lowest frequency ν_1 is called the fundamental frequency. It produces the fundamental mode of vibration, or else, the first harmonic. Twice the fundamental frequency or the second harmonic ν_2 produces the pattern shown in figure 1, and so on.

The resonance frequencies ν_n are related to the wave velocity in the string *v* and to the length of the string L:

$$L = n \cdot \frac{\lambda_n}{2}, \tag{4}$$

and,

$$\nu_n = \frac{v}{\lambda_n} \qquad \lambda_n = \frac{v}{\nu_n}, \tag{5}$$

resulting that: $\nu_n = nv/2L$ and since the wave velocity is $v = \sqrt{F/\mu}$, then the fundamental frequency of the stretched string is:

$$\nu_1 = \frac{v}{2L} = \frac{1}{2L} \cdot \sqrt{\frac{T}{\mu}}, \tag{6}$$

where $T \equiv F$ is the tension in the string and μ is the linear density of the string.

4. Applications

The standing waves can be often observed in musical instruments with cords. Here, the vibrations are produced with a bow by pinching or hitting. In this way the week sound and with no musicality of the vibrating component is amplified and enriched with resonance frequencies, of which spectra create a timbre specific to each instrument. In the end, a sound wave is created of which propagation in the resonance cavity produces the standing waves.

Depending on the propagation media geometry, the standing wave can be one-, bi- or tri-dimensional, as example: i) for one-dimension we have: the vibrating cord, vibrating tube; ii) for two dimension we have: the resonance box of musical instruments, the surface of a liquid, vibrating membrane as those of a drum; iii) for three dimensions we have the interior of a room, instrument, etc.

5. Experimental set-up and procedure

The string is laid between a fixed hook and a body with mass m which hangs at the other end. The important geometrical properties of string are: L the length, d the diameter of the wire and ρ the mass density of the material. A magnet piece excites the transverse oscillations at a point of the string, due to the alternative electrical current which is propagating along the string. The terminals of a variable frequency generator are connected to the ends of the string (see figure 2).

The problems to follow are:

1. to measure the fundamental frequency ν_1 of the string and to observe its dependence on the force which tensions the string.
2. to measure the superior harmonic frequencies ($\nu_2, \nu_3, \nu_4 ...$) and the corresponding wavelengths of the standing waves.
3. to compare the theoretical frequencies to the experimental values obtained.
4. data are to be recorded in the table below.

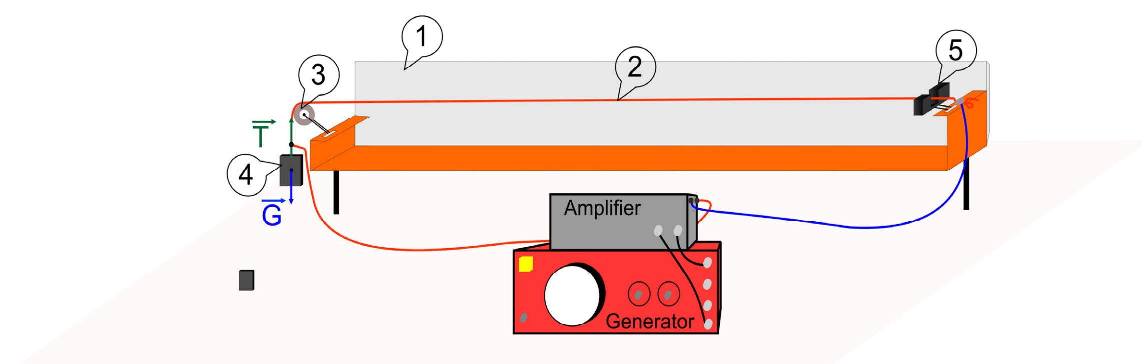

Figure 2.

Table 1

m [kg]	$T = mg$ [N]	$v = \sqrt{\frac{T}{\mu}}$ [m/s]	ν_n [Hz]	1	2	3	4	5	λ_n [m]	1	2	3	4	5
			ν_{calc}						λ_{calc}					
			ν_{exp}						λ_{exp}					
			ν_{calc}						λ_{calc}					
			ν_{exp}						λ_{exp}					

6. Data processing:

1. the tension in the string is due to the weight of body hang at the end of the string: $T = mg$;
2. the theoretical value of transverse waves velocity: $v_t = \sqrt{T/\mu}$;
3. the wavelength of standing waves: $\lambda_n = \frac{2L}{n}$;
4. the experimental velocity of standing transverse waves: $v_{exp} = \lambda_n \nu_n$;
5. the relative deviations:

$$\frac{\Delta v_{exp}}{v_{exp}} = \frac{\Delta L}{L} + \frac{\Delta \nu}{\nu}$$
$$\frac{\Delta v_t}{v_t} = \frac{\Delta d}{d} + \frac{1}{2} \cdot \left(\frac{\Delta m}{m} + \frac{\Delta g}{g} + \frac{\Delta \rho}{\rho} \right), \qquad (7)$$

where *d* is the diameter of the wire; *m* is the mass of the body hang at the end of the string; *g* is the gravitational acceleration and ρ is the density of the material.

7. Longitudinal waves. Theoretical approach

The longitudinal waves can be transmitted in all media, not only in solids. Sound is a longitudinal mechanical wave which can also be transmitted in air. The velocity of longitudinal waves was deduced by Newton as: $v = \sqrt{E/\rho}$, where E represents the Young's modulus of elasticity and ρ is the mass density of the elastic material through which the wave are propagates.

The longitudinal elongation $\psi(x,t)$ of a spring loop located at the distance x is given by the equation of the progressive wave: $\psi_p = A \cdot \sin\left[\omega\left(t - \frac{x}{v}\right)\right]$. The equation of the reflected wave at the same point at a distance x from the source is: $\psi_r = A\cdot\sin\left[\omega\left(t+\frac{x}{v}\right)+\pi\right]$. The resulted wave will be:

$$\psi = \psi_p + \psi_r = 2A\cdot\cos\left(\frac{\pi}{2}-\frac{\omega}{v}x\right)\cdot\sin\left(\omega t-\frac{\pi}{2}\right) = 2A\cdot\sin\left(\frac{\omega}{v}x\right)\cdot\cos(\omega t). \quad (8)$$

At any time, the amplitude of standing waves has minimum values if $2A\cdot\sin\left(\frac{\omega}{v}x\right)=0$. This corresponds to $\frac{\omega}{v}x = n\pi \leftrightarrow \frac{2\pi}{T}\frac{x}{v} = n\pi \rightarrow x_m = n\frac{\lambda}{2}$.

The amplitude takes maximum values if $\sin\left(\frac{\omega}{v}x\right)=\pm 1$, for $\frac{\omega}{v}x = (2n+1)\frac{\pi}{2} \leftrightarrow x_m = (2n+1)\frac{\lambda}{4}$.

Therefore, each coil of the spring performs harmonic oscillations with the same angular frequency ω and with amplitudes which depend on the distance from the point which vibrates to the end of the spring according to the eq. (8). When *n* antinodes exist along the spring, the length of the spring and the wavelength of the longitudinal wave fulfill the relationship: $L = n\lambda_n/2$ and the wavelength of the n^{th} mode of vibration is: $\lambda_n = 2L/n$. The corresponding eigen frequencies of the spring system are: $\nu_n = v/\lambda_n = nv/2L$, then $\nu_n = n\nu_1$, where ν_1 is the fundamental frequency of the spring.

There is one more remark concerning the studied system: the stretched spring represents a quasi-continuous medium of propagation and the Newton's formula for the speed of longitudinal waves in elastic media becomes: $v = \sqrt{\langle E\rangle/\langle\rho\rangle}$, where $\langle E\rangle$ represents the Young's modulus of elasticity for the stretched spring and $\langle\rho\rangle$ is the density of the spring viewed as a quasi-continuous medium of propagation. If:

$$\langle E\rangle = \frac{F}{S}\frac{L}{\Delta L} = \frac{k\Delta L}{S}\frac{L}{\Delta L} = \frac{kL}{S}, \quad (9)$$

and:

$$\langle\rho\rangle = \frac{m}{V} = \frac{m}{SL}, \quad (10)$$

then:

$$\nu_n = \frac{n}{2L}\sqrt{\frac{\langle E\rangle}{\langle\rho\rangle}} = \frac{n}{2}\sqrt{\frac{k}{m}}. \quad (11)$$

8. Experimental set-up and procedure

To observe the standing waves in a spring, longitudinal vibrations are excited by a rigid connection of one end of the spring to the membrane of a loudspeaker. The sockets of a loudspeaker have to be connected to the terminals of the alternative current generator. The loudspeaker transforms the alternative electrical signal into mechanical vibrations. The longitudinal waves transmitted along the spring are reflected at the fixed end. The regressive waves produced in this way interfere with the direct waves. Standing waves are observed in the spring when the frequency of the electrical signal (and that of the membrane) is equal to one of the characteristic frequencies of the spring system (resonance phenomena).

The purpose of this experiment is to observe and study the longitudinal standing waves induced in the spring. The experimental frequencies corresponding to the modes of vibration (n = 2–8) are to be recorded in the data table below.

Table 2

n	1	2	3	4	5	6	7	8
ν_n^{exp} (Hz)								
ν_n^{theor} (Hz)								

9. Data processing:

1. calculate the frequencies of the first 8 vibration modes of the spring using the equation 11.
2. plot the graph of $\nu_n^{exp} = f\left(\nu_n^{theor}\right)$ (only the points, which have not to be connected)
3. the speed of longitudinal waves in the spring system: $v = L \cdot \sqrt{k/m}$.
4. the speed found by experimental determinations: $v = \lambda_n \nu_n$;
5. the mass of the spring m = 97 g; the length L = 59 cm, the diameter of the loops of the spring d = 14.8 mm and the spring elastic constant k = 76.05 N/m.
6. finally, the students have to find a method to determine by themselves the frequency of the fundamental mode of vibration.